NAG C Library Function Document

nag_zgerfs (f07avc)

1 Purpose

nag_zgerfs (f07avc) returns error bounds for the solution of a complex system of linear equations with multiple right-hand sides, $AX = B$, $A^T X = B$ or $A^H X = B$. It improves the solution by iterative refinement, in order to reduce the backward error as much as possible.

2 Specification

```
void nag_zgerfs (Nag_OrderType order, Nag_TransType trans, Integer n, Integer nrhs,
      const Complex a[], Integer pda, const Complex af[], Integer pdaf,
      const Integer ipiv[], const Complex b[], Integer pdb, Complex x[],
      Integer pdx, double ferr[], double berr[], NagError *fail)
```

3 Description

nag_zgerfs (f07avc) returns the backward errors and estimated bounds on the forward errors for the solution of a complex system of linear equations with multiple right-hand sides $AX = B$, $A^T X = B$ or $A^H X = B$. The function handles each right-hand side vector (stored as a column of the matrix B) independently, so we describe the function of nag_zgerfs (f07avc) in terms of a single right-hand side b and solution x.

Given a computed solution x, the function computes the *component-wise backward error* β. This is the size of the smallest relative perturbation in each element of A and b such that x is the exact solution of a perturbed system

$$(A + \delta A)x = b + \delta b$$
$$|\delta a_{ij}| \leq \beta |a_{ij}| \quad \text{and} \quad |\delta b_i| \leq \beta |b_i|.$$

Then the function estimates a bound for the *component-wise forward error* in the computed solution, defined by:

$$\max_i |x_i - \hat{x}_i| / \max_i |x_i|$$

where $\hat{x}$ is the true solution.

For details of the method, see the f07 Chapter Introduction.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: **order** – Nag_OrderType *Input*

On entry: the **order** parameter specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = **Nag_RowMajor**. See Section 2.2.1.4 of the Essential Introduction for a more detailed explanation of the use of this parameter.

Constraint: **order** = **Nag_RowMajor** or **Nag_ColMajor**.

2: **trans** – Nag_TransType *Input*

On entry: indicates the form of the linear equations for which X is the computed solution as follows:

if **trans** $=$ **Nag_NoTrans**, the linear equations are of the form $AX = B$;

if **trans** $=$ **Nag_Trans**, the linear equations are of the form $A^T X = B$;

if **trans** $=$ **Nag_ConjTrans**, the linear equations are of the form $A^H X = B$.

Constraint: **trans** $=$ **Nag_NoTrans**, **Nag_Trans** or **Nag_ConjTrans**.

3: **n** – Integer *Input*

On entry: n, the order of the matrix A.

Constraint: **n** ≥ 0.

4: **nrhs** – Integer *Input*

On entry: r, the number of right-hand sides.

Constraint: **nrhs** ≥ 0.

5: **a**$[dim]$ – const Complex *Input*

Note: the dimension, dim, of the array **a** must be at least $\max(1, \mathbf{pda} \times \mathbf{n})$.

If **order** $=$ **Nag_ColMajor**, the (i, j)th element of the matrix A is stored in $\mathbf{a}[(j-1) \times \mathbf{pda} + i - 1]$ and if **order** $=$ **Nag_RowMajor**, the (i, j)th element of the matrix A is stored in $\mathbf{a}[(i-1) \times \mathbf{pda} + j - 1]$.

On entry: the n by n original matrix A as supplied to nag_zgetrf (f07arc).

6: **pda** – Integer *Input*

On entry: the stride separating matrix row or column elements (depending on the value of **order**) in the array **a**.

Constraint: **pda** $\geq \max(1, \mathbf{n})$.

7: **af**$[dim]$ – const Complex *Input*

Note: the dimension, dim, of the array **af** must be at least $\max(1, \mathbf{pdaf} \times \mathbf{n})$.

If **order** $=$ **Nag_ColMajor**, the (i, j)th element of the matrix is stored in $\mathbf{af}[(j-1) \times \mathbf{pdaf} + i - 1]$ and if **order** $=$ **Nag_RowMajor**, the (i, j)th element of the matrix is stored in $\mathbf{af}[(i-1) \times \mathbf{pdaf} + j - 1]$.

On entry: the LU factorization of A, as returned by nag_zgetrf (f07arc).

8: **pdaf** – Integer *Input*

On entry: the stride separating matrix row or column elements (depending on the value of **order**) in the array **af**.

Constraint: **pdaf** $\geq \max(1, \mathbf{n})$.

9: **ipiv**$[dim]$ – const Integer *Input*

Note: the dimension, dim, of the array **ipiv** must be at least $\max(1, \mathbf{n})$.

On entry: the pivot indices, as returned by nag_zgetrf (f07arc).

10: **b**$[dim]$ – const Complex *Input*

Note: the dimension, dim, of the array **b** must be at least $\max(1, \mathbf{pdb} \times \mathbf{nrhs})$ when **order** $=$ **Nag_ColMajor** and at least $\max(1, \mathbf{pdb} \times \mathbf{n})$ when **order** $=$ **Nag_RowMajor**.

If **order** $=$ **Nag_ColMajor**, the (i, j)th element of the matrix B is stored in $\mathbf{b}[(j-1) \times \mathbf{pdb} + i - 1]$ and if **order** $=$ **Nag_RowMajor**, the (i, j)th element of the matrix B is stored in $\mathbf{b}[(i-1) \times \mathbf{pdb} + j - 1]$.

On entry: the n by r right-hand side matrix B.

11: **pdb** – Integer *Input*

On entry: the stride separating matrix row or column elements (depending on the value of **order**) in the array **b**.

Constraints:

if **order** = **Nag_ColMajor**, **pdb** $\geq \max(1, \mathbf{n})$;
if **order** = **Nag_RowMajor**, **pdb** $\geq \max(1, \mathbf{nrhs})$.

12: **x**[dim] – Complex *Input/Output*

Note: the dimension, dim, of the array **x** must be at least $\max(1, \mathbf{pdx} \times \mathbf{nrhs})$ when **order** = **Nag_ColMajor** and at least $\max(1, \mathbf{pdx} \times \mathbf{n})$ when **order** = **Nag_RowMajor**.

If **order** = **Nag_ColMajor**, the (i, j)th element of the matrix X is stored in $\mathbf{x}[(j - 1) \times \mathbf{pdx} + i - 1]$ and if **order** = **Nag_RowMajor**, the (i, j)th element of the matrix X is stored in $\mathbf{x}[(i - 1) \times \mathbf{pdx} + j - 1]$.

On entry: the n by r solution matrix X, as returned by nag_zgetrs (f07asc).

On exit: the improved solution matrix X.

13: **pdx** – Integer *Input*

On entry: the stride separating matrix row or column elements (depending on the value of **order**) in the array **x**.

Constraints:

if **order** = **Nag_ColMajor**, **pdx** $\geq \max(1, \mathbf{n})$;
if **order** = **Nag_RowMajor**, **pdx** $\geq \max(1, \mathbf{nrhs})$.

14: **ferr**[dim] – double *Output*

Note: the dimension, dim, of the array **ferr** must be at least $\max(1, \mathbf{nrhs})$.

On exit: $\mathbf{ferr}[j - 1]$ contains an estimated error bound for the jth solution vector, that is, the jth column of X, for $j = 1, 2, \ldots, r$.

15: **berr**[dim] – double *Output*

Note: the dimension, dim, of the array **berr** must be at least $\max(1, \mathbf{nrhs})$.

On exit: $\mathbf{berr}[j - 1]$ contains the component-wise backward error bound β for the jth solution vector, that is, the jth column of X, for $j = 1, 2, \ldots, r$.

16: **fail** – NagError * *Output*

The NAG error parameter (see the Essential Introduction).

6 Error Indicators and Warnings

NE_INT

On entry, **n** = ⟨*value*⟩.
Constraint: $\mathbf{n} \geq 0$.

On entry, **nrhs** = ⟨*value*⟩.
Constraint: $\mathbf{nrhs} \geq 0$.

On entry, **pda** = ⟨*value*⟩.
Constraint: $\mathbf{pda} > 0$.

On entry, **pdaf** = ⟨*value*⟩.
Constraint: $\mathbf{pdaf} > 0$.

On entry, **pdb** $= \langle value \rangle$.
Constraint: **pdb** > 0.

On entry, **pdx** $= \langle value \rangle$.
Constraint: **pdx** > 0.

NE_INT_2

On entry, **pda** $= \langle value \rangle$, **n** $= \langle value \rangle$.
Constraint: **pda** $\geq \max(1, \mathbf{n})$.

On entry, **pdaf** $= \langle value \rangle$, **n** $= \langle value \rangle$.
Constraint: **pdaf** $\geq \max(1, \mathbf{n})$.

On entry, **pdb** $= \langle value \rangle$, **n** $= \langle value \rangle$.
Constraint: **pdb** $\geq \max(1, \mathbf{n})$.

On entry, **pdb** $= \langle value \rangle$, **nrhs** $= \langle value \rangle$.
Constraint: **pdb** $\geq \max(1, \mathbf{nrhs})$.

On entry, **pdx** $= \langle value \rangle$, **n** $= \langle value \rangle$.
Constraint: **pdx** $\geq \max(1, \mathbf{n})$.

On entry, **pdx** $= \langle value \rangle$, **nrhs** $= \langle value \rangle$.
Constraint: **pdx** $\geq \max(1, \mathbf{nrhs})$.

NE_ALLOC_FAIL

Memory allocation failed.

NE_BAD_PARAM

On entry, parameter $\langle value \rangle$ had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 Accuracy

The bounds returned in **ferr** are not rigorous, because they are estimated, not computed exactly; but in practice they almost always overestimate the actual error.

8 Further Comments

For each right-hand side, computation of the backward error involves a minimum of $16n^2$ real floating-point operations. Each step of iterative refinement involves an additional $24n^2$ real operations. At most 5 steps of iterative refinement are performed, but usually only 1 or 2 steps are required.

Estimating the forward error involves solving a number of systems of linear equations of the form $Ax = b$ or $A^H x = b$; the number is usually 5 and never more than 11. Each solution involves approximately $8n^2$ real operations.

The real analogue of this function is nag_dgerfs (f07ahc).

9 Example

To solve the system of equations $AX = B$ using iterative refinement and to compute the forward and backward error bounds, where

$$A = \begin{pmatrix} -1.34+2.55i & 0.28+3.17i & -6.39-2.20i & 0.72-0.92i \\ -0.17-1.41i & 3.31-0.15i & -0.15+1.34i & 1.29+1.38i \\ -3.29-2.39i & -1.91+4.42i & -0.14-1.35i & 1.72+1.35i \\ 2.41+0.39i & -0.56+1.47i & -0.83-0.69i & -1.96+0.67i \end{pmatrix}$$

and

$$B = \begin{pmatrix} 26.26+51.78i & 31.32-\ \ 6.70i \\ 6.43-\ \ 8.68i & 15.86-\ \ 1.42i \\ -5.75+25.31i & -2.15+30.19i \\ 1.16+\ \ 2.57i & -2.56+\ \ 7.55i \end{pmatrix}.$$

Here A is nonsymmetric and must first be factorized by nag_zgetrf (f07arc).

9.1 Program Text

```
/* nag_zgerfs (f07avc) Example Program.
 *
 * Copyright 2001 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf07.h>
#include <nagx04.h>

int main(void)
{
  /* Scalars */
  Integer  berr_len, i, ferr_len, ipiv_len, j, n, nrhs;
  Integer  pda, pdaf, pdb, pdx;
  Integer  exit_status=0;
  NagError fail;
  Nag_OrderType order;
  /* Arrays */
  Complex *a=0, *af=0, *b=0, *x=0;
  double  *berr=0, *ferr=0;
  Integer *ipiv=0;

#ifdef NAG_COLUMN_MAJOR
#define A(I,J) a[(J-1)*pda + I - 1]
#define AF(I,J) af[(J-1)*pdaf + I - 1]
#define B(I,J) b[(J-1)*pdb + I - 1]
#define X(I,J) x[(J-1)*pdx + I - 1]
  order = Nag_ColMajor;
#else
#define A(I,J) a[(I-1)*pda + J - 1]
#define AF(I,J) af[(I-1)*pdaf + J - 1]
#define B(I,J) b[(I-1)*pdb + J - 1]
#define X(I,J) x[(I-1)*pdx + J - 1]
  order = Nag_RowMajor;
#endif

  INIT_FAIL(fail);
  Vprintf("f07avc Example Program Results\n\n");

  /* Skip heading in data file */
  Vscanf("%*[^\n] ");
  Vscanf("%ld%ld%*[^\n] ", &n, &nrhs);
#ifdef NAG_COLUMN_MAJOR
  pda = n;
  pdaf = n;
  pdb = n;
  pdx = n;
```

```
#else
  pda = n;
  pdaf = n;
  pdb = nrhs;
  pdx = nrhs;
#endif
  ipiv_len = n;
  ferr_len = n;
  berr_len = nrhs;

  /* Allocate memory */
  if ( !(a = NAG_ALLOC(n * n, Complex)) ||
       !(af = NAG_ALLOC(n * n, Complex)) ||
       !(b = NAG_ALLOC(n * nrhs, Complex)) ||
       !(x = NAG_ALLOC(n * n, Complex)) ||
       !(berr = NAG_ALLOC(berr_len, double)) ||
       !(ferr = NAG_ALLOC(ferr_len, double)) ||
       !(ipiv = NAG_ALLOC(ipiv_len, Integer)) )
    {
      Vprintf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }

  /* Read A and B from data file, and copy A to AF and B to X */
  for (i = 1; i <= n; ++i)
    {
      for (j = 1; j <= n; ++j)
        Vscanf(" ( %lf , %lf )", &A(i,j).re, &A(i,j).im);
    }
  Vscanf("%*[^\n] ");
  for (i = 1; i <= n; ++i)
    {
      for (j = 1; j <= nrhs; ++j)
        Vscanf(" ( %lf , %lf )", &B(i,j).re, &B(i,j).im);
    }
  Vscanf("%*[^\n] ");

  for (i = 1; i <= n; ++i)
    {
      for (j = 1; j <= n; ++j)
        {
          AF(i,j).re = A(i,j).re;
          AF(i,j).im = A(i,j).im;
        }
    }
  for (i = 1; i <= n; ++i)
    {
      for (j = 1; j <= nrhs; ++j)
        {
          X(i,j).re = B(i,j).re;
          X(i,j).im = B(i,j).im;
        }
    }
  /* Factorize A in the array AF */
  f07arc(order, n, n, af, pdaf, ipiv, &fail);
  if (fail.code != NE_NOERROR)
    {
      Vprintf("Error from f07arc.\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }
  /* Compute solution in the array X */
  f07asc(order, Nag_NoTrans, n, nrhs, af, pdaf, ipiv, x, pdx, &fail);
  if (fail.code != NE_NOERROR)
    {
      Vprintf("Error from f07asc.\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }
  /* Improve solution, and compute backward errors and */
```

```
  /* estimated bounds on the forward errors */
  f07avc(order, Nag_NoTrans, n, nrhs, a, pda, af, pdaf, ipiv, b, pdb, x,
         pdx, ferr, berr, &fail);
  if (fail.code != NE_NOERROR)
    {
      Vprintf("Error from f07avc.\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }

  /* Print solution */
  x04dbc(order, Nag_GeneralMatrix, Nag_NonUnitDiag, n, nrhs, x, pdx,
         Nag_BracketForm, "%7.4f", "Solution(s)", Nag_IntegerLabels,
         0, Nag_IntegerLabels, 0, 80, 0, 0, &fail);
  if (fail.code != NE_NOERROR)
    {
      Vprintf("Error from x04dbc.\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }

  Vprintf("\nBackward errors (machine-dependent)\n");

  for (j = 1; j <= nrhs; ++j)
    Vprintf("%11.1e%s", berr[j-1], j%4==0 ?"\n":" ");
  Vprintf("\nEstimated forward error bounds (machine-dependent)\n");
  for (j = 1; j <= nrhs; ++j)
    Vprintf("%11.1e%s", ferr[j-1], j%4==0 || j==nrhs ?"\n":" ");
 END:
  if (a) NAG_FREE(a);
  if (af) NAG_FREE(af);
  if (b) NAG_FREE(b);
  if (x) NAG_FREE(x);
  if (berr) NAG_FREE(berr);
  if (ferr) NAG_FREE(ferr);
  if (ipiv) NAG_FREE(ipiv);
  return exit_status;
}
```

9.2 Program Data

```
f07avc Example Program Data
  4  2                                                        :Values of N and NRHS
 (-1.34, 2.55) ( 0.28, 3.17) (-6.39,-2.20) ( 0.72,-0.92)
 (-0.17,-1.41) ( 3.31,-0.15) (-0.15, 1.34) ( 1.29, 1.38)
 (-3.29,-2.39) (-1.91, 4.42) (-0.14,-1.35) ( 1.72, 1.35)
 ( 2.41, 0.39) (-0.56, 1.47) (-0.83,-0.69) (-1.96, 0.67)  :End of matrix A
 (26.26, 51.78) (31.32, -6.70)
 ( 6.43, -8.68) (15.86, -1.42)
 (-5.75, 25.31) (-2.15, 30.19)
 ( 1.16,  2.57) (-2.56,  7.55)                            :End of matrix B
```

9.3 Program Results

```
f07avc Example Program Results

 Solution(s)
                     1                   2
 1  ( 1.0000, 1.0000)  (-1.0000,-2.0000)
 2  ( 2.0000,-3.0000)  ( 5.0000, 1.0000)
 3  (-4.0000,-5.0000)  (-3.0000, 4.0000)
 4  ( 0.0000, 6.0000)  ( 2.0000,-3.0000)

Backward errors (machine-dependent)
    9.1e-17     7.2e-17
Estimated forward error bounds (machine-dependent)
    5.9e-14     7.6e-14
```